Lecture 5: Fundamentals Part 4 Open-Channel Flow 2

## WMD651: Water Resources Systems Design

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Non-Uniform Flow

- Open channels do not always exhibit uniform flow

- $y < y_n$ and $y > y_n$ are both possible depending on boundary conditions (BCs)

- Boundary conditions:
  - Represent physical constraints
  - May fix y, Q, or both at either end of a channel
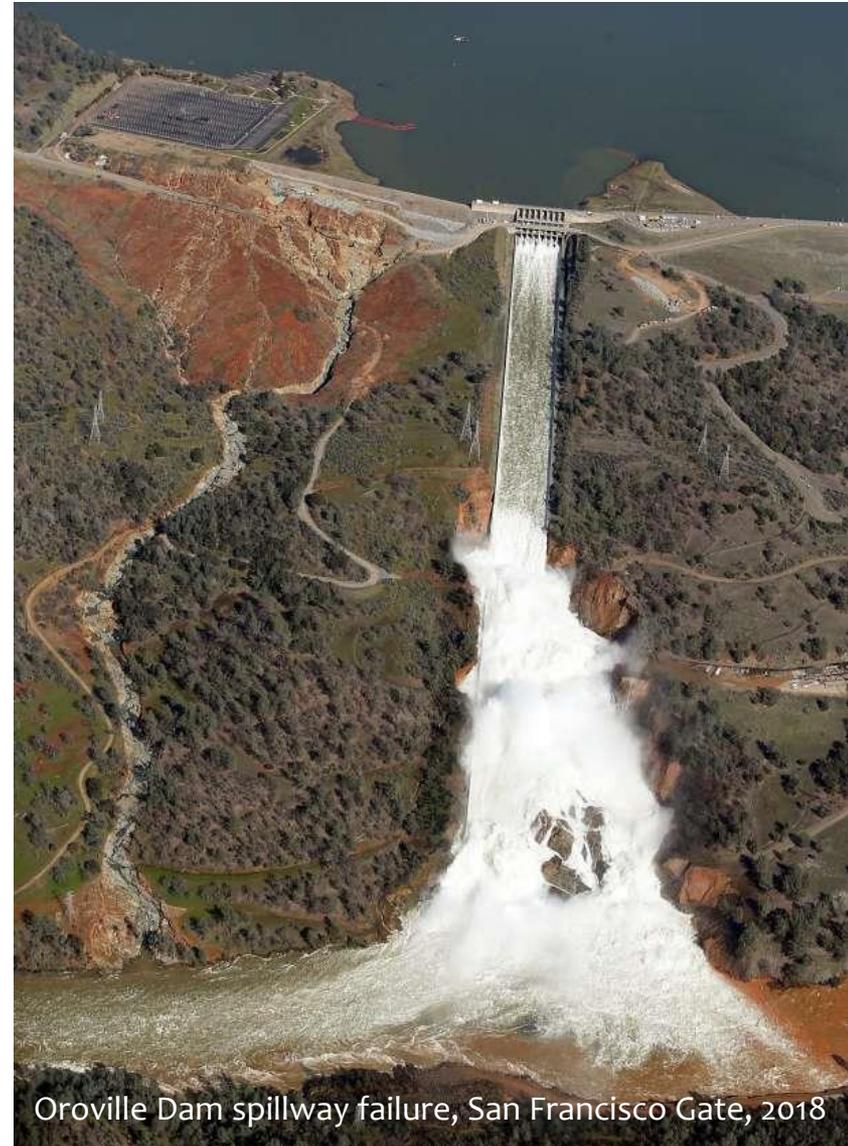
Oroville Dam spillway failure, San Francisco Gate, 2018

In this lesson…

- Characteristics of non-uniform flow
- Critical flow:
    - What is critical flow, and why is it important?
    - Froude number
    - Example – super- and sub-critical flow
- Specific energy
- Hydraulic jumps
- Non-uniform flow profiles
    - Water surface profile classification
    - Example – labeling and sketching flow profiles

Froude Number and Critical Flow

- Critical flow:
  - Critical flow occurs when E at minimum for given Q
  - Specific energy: $E = y + V^2/2g$
  - Corresponding depth is known as **critical depth**, y_c
  - Super critical flow: $y < y_c$ (shallower flow)
  - Sub-critical flow: $y > y_c$ (deeper flow)
- Froude number:
  - Represents ratio of flow velocity to wave speed
  - Describes whether flow is critical $(Fr = 1)$, super critical $(Fr > 1)$, sub-critical $(Fr < 1)$
  - Equation: $\boxed{Fr = \sqrt{V^2/gD}}$
  - Hydraulic depth: $D = A/T$ → $\boxed{Fr = \sqrt{\dfrac{Q^2 T}{g A^3}}}$
  - Note: T = top width of flow (m)

Example 1 – Froude Number

- **Problem:** A circular conduit is characterized by $D = 375$ mm and $y = 150$ mm. Determine whether flow is super- or sub-critical for a flow of $Q = 0.055$ m³/s.

Example 1 – Froude Number

- **Problem:** A circular conduit is characterized by D = 375 mm and y = 150 mm. Determine whether flow is super- or sub-critical for a flow of Q = 0.055 m³/s.

- **Solution:**
  - Approach:
    - First determine θ using y and D
    - Using θ, determine the flow area A and top width of the flow section, T
    - Use continuity to determine the flow velocity V
    - Lastly, calculate the Froude number
  - Relative depth: $y/D = (150\ mm)/(375\ mm) = 0.4$

Example 1 – Froude Number

- **Solution (continued):**
  - Using the table below, θ = 2.74 rad for y/D = 0.40
  - Relative flow area: $A/A_{full} = 0.374$

Theta (rad)	y/D	a/A	r/R	q/Q
0.000	0.00	0.000	0.000	0.000
0.902	0.05	0.019	0.130	0.005
1.287	0.10	0.052	0.254	0.021
1.591	0.15	0.094	0.372	0.049
1.855	0.20	0.142	0.482	0.088
2.094	0.25	0.196	0.587	0.137
2.319	0.30	0.252	0.684	0.196
2.532	0.35	0.312	0.774	0.263
2.739	0.40	0.374	0.857	0.337
2.941	0.45	0.436	0.932	0.417
3.142	0.50	0.500	1.000	0.500

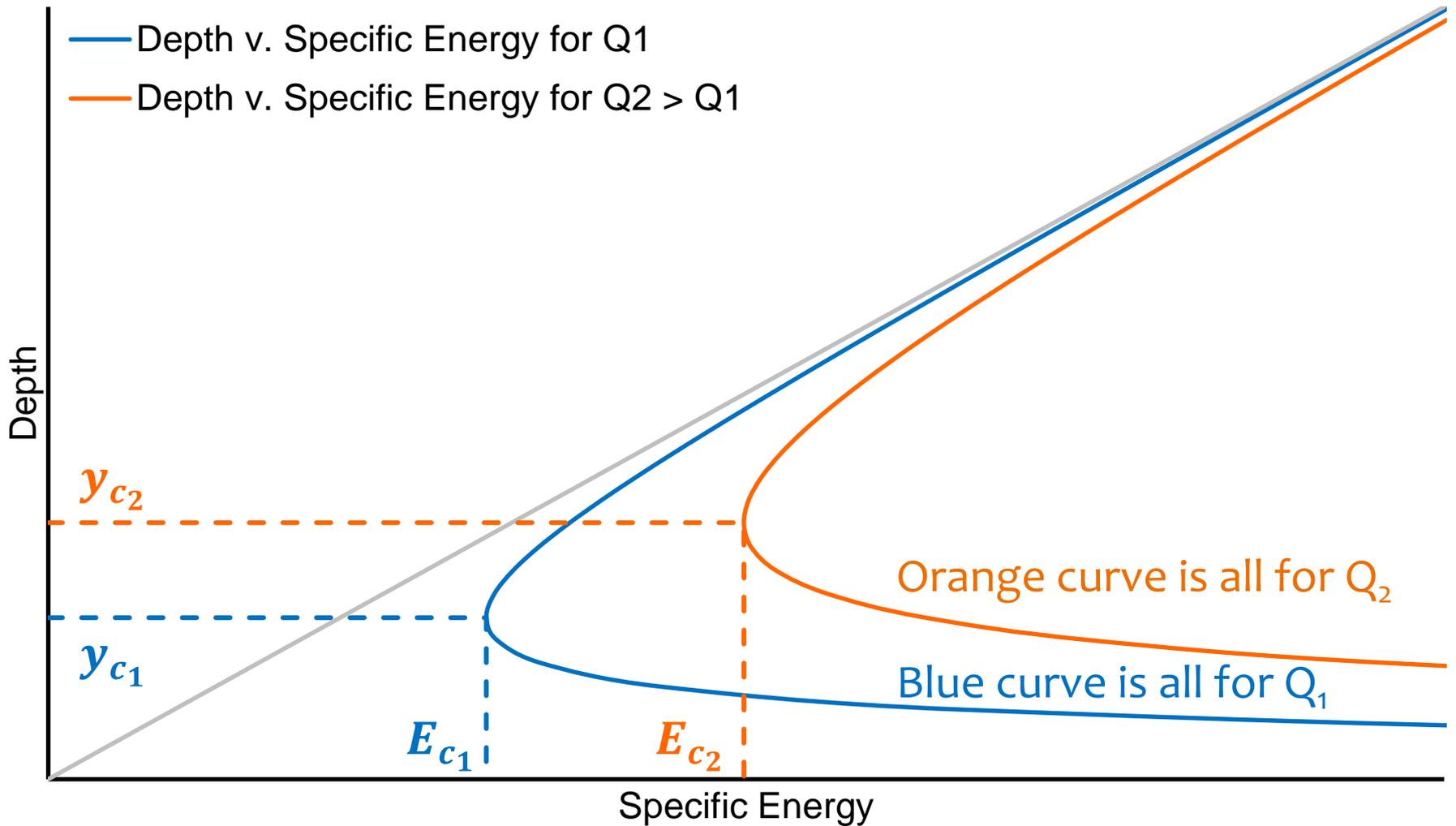| Theta (rad)	y/D	a/A	r/R	q/Q
3.342	0.55	0.564	1.060	0.586
3.544	0.60	0.626	1.111	0.672
3.751	0.65	0.688	1.153	0.756
3.965	0.70	0.748	1.185	0.837
4.189	0.75	0.804	1.207	0.912
4.429	0.80	0.858	1.217	0.977
4.692	0.85	0.906	1.213	1.030
4.996	0.90	0.948	1.192	1.066
5.381	0.95	0.981	1.146	1.075
6.271	1.00	1.000	1.002	1.001

Example 1 – Froude Number

- **Solution (continued):**
  - Full flow area: $A_{full} = \frac{1}{4} \pi D^2 = 0.110 \; m^2$
  - Actual flow area: $A = (A/A_{full}) \times A_{full} = 0.0413 \; m^2$
  - Top width:
    - For a circular section, $T = D \times \sin(\theta/2)$
    - $T = D \times \sin(\theta/2)$
      $= (375 \; mm) \times \sin(2.74/2)$
      $= 367 \; mm$
  - Flow velocity: $V = \frac{Q}{A} = \frac{0.055 \; m^3/s}{0.0413 \; m^2} = 1.33 \; m/s$
  - Froude number: $Fr = \sqrt{\dfrac{Q^2 T}{g A^3}} = \sqrt{\dfrac{(0.055 m^3/s)^2 (0.367 \; m)}{(9.81 \; m/s^2)(0.0413 \; m^2)^3}} = 1.27$
  - Because $Fr > 1$, the flow conditions are super critical

Minimum Specific Energy
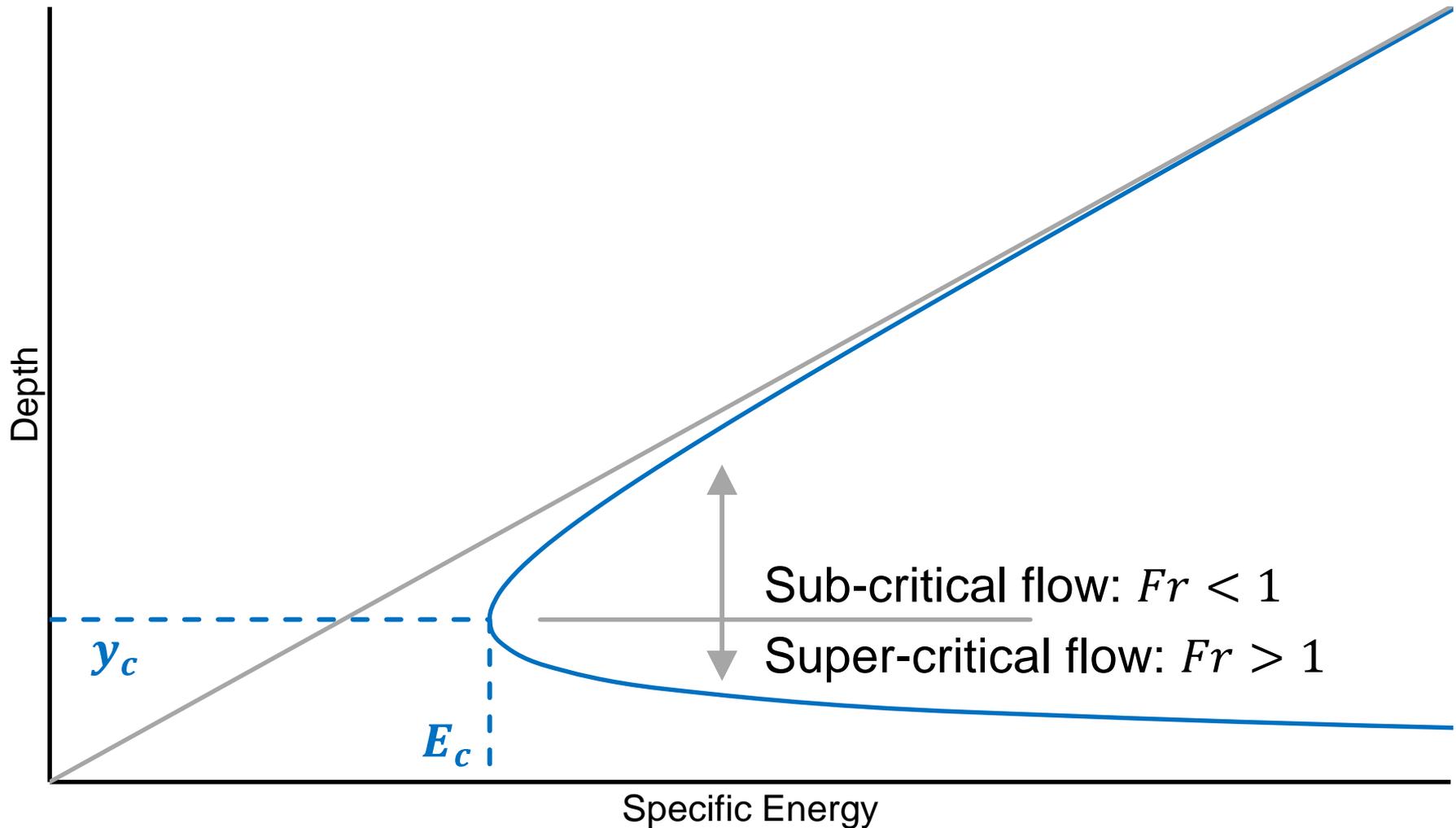
Min. Specific Energy, Different Q

Depth v. Specific Energy for Q1
Depth v. Specific Energy for Q2 > Q1

y_{c_2}

y_{c_1}

E_{c_1}

E_{c_2}
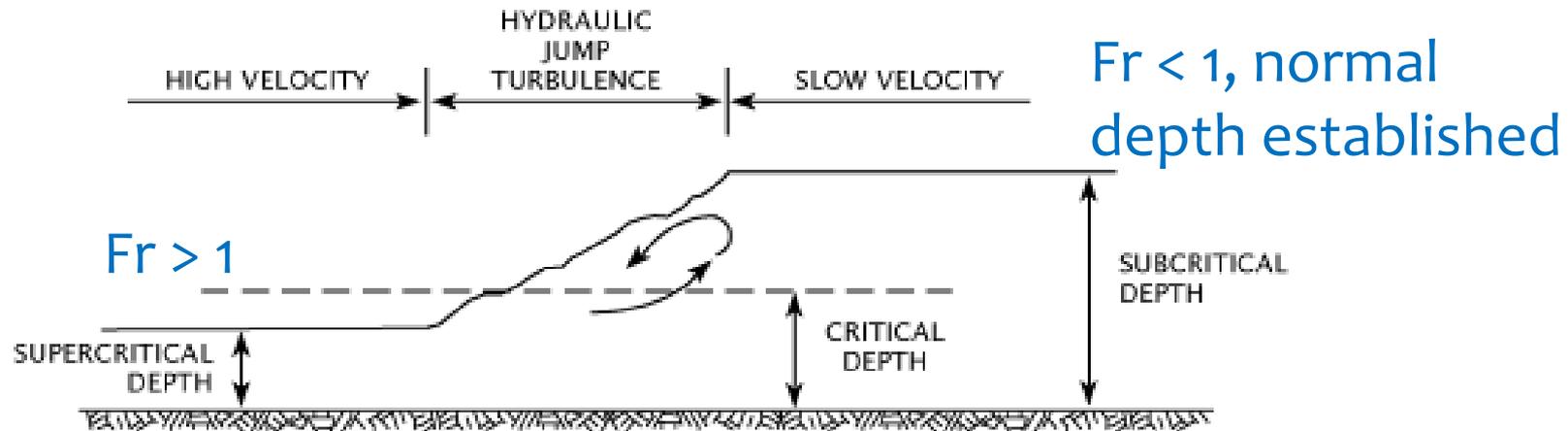
Orange curve is all for Q_2

Blue curve is all for Q_1

Depth

Specific Energy
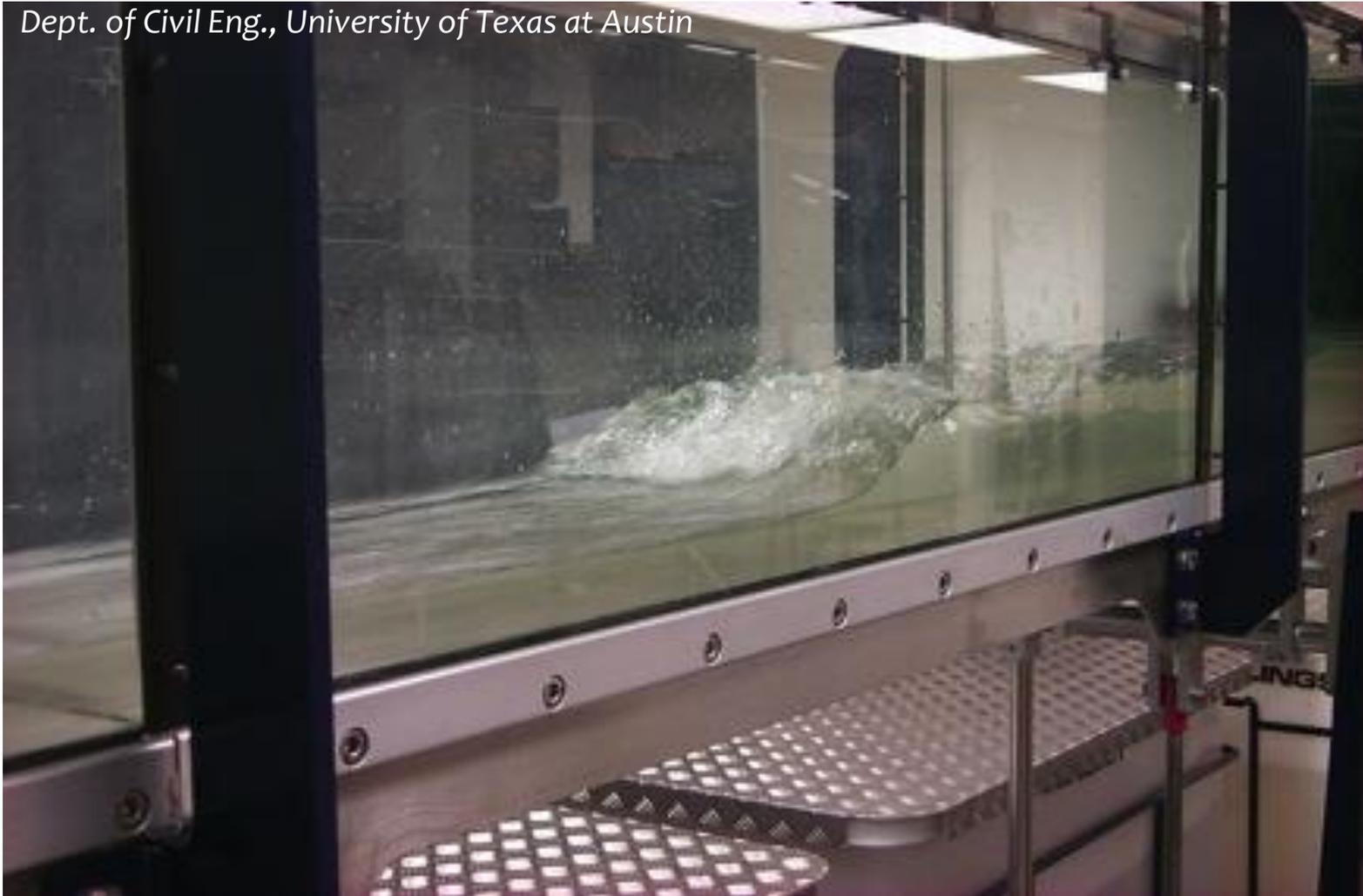
Why is critical flow important?

- Critical flow arises at certain boundary conditions (e.g., restrictions, weirs)

- Flow conditions with Fr near 1 are often unstable → hydraulic jumps can arise as flow transitions from super to sub-critical (or vice-versa)
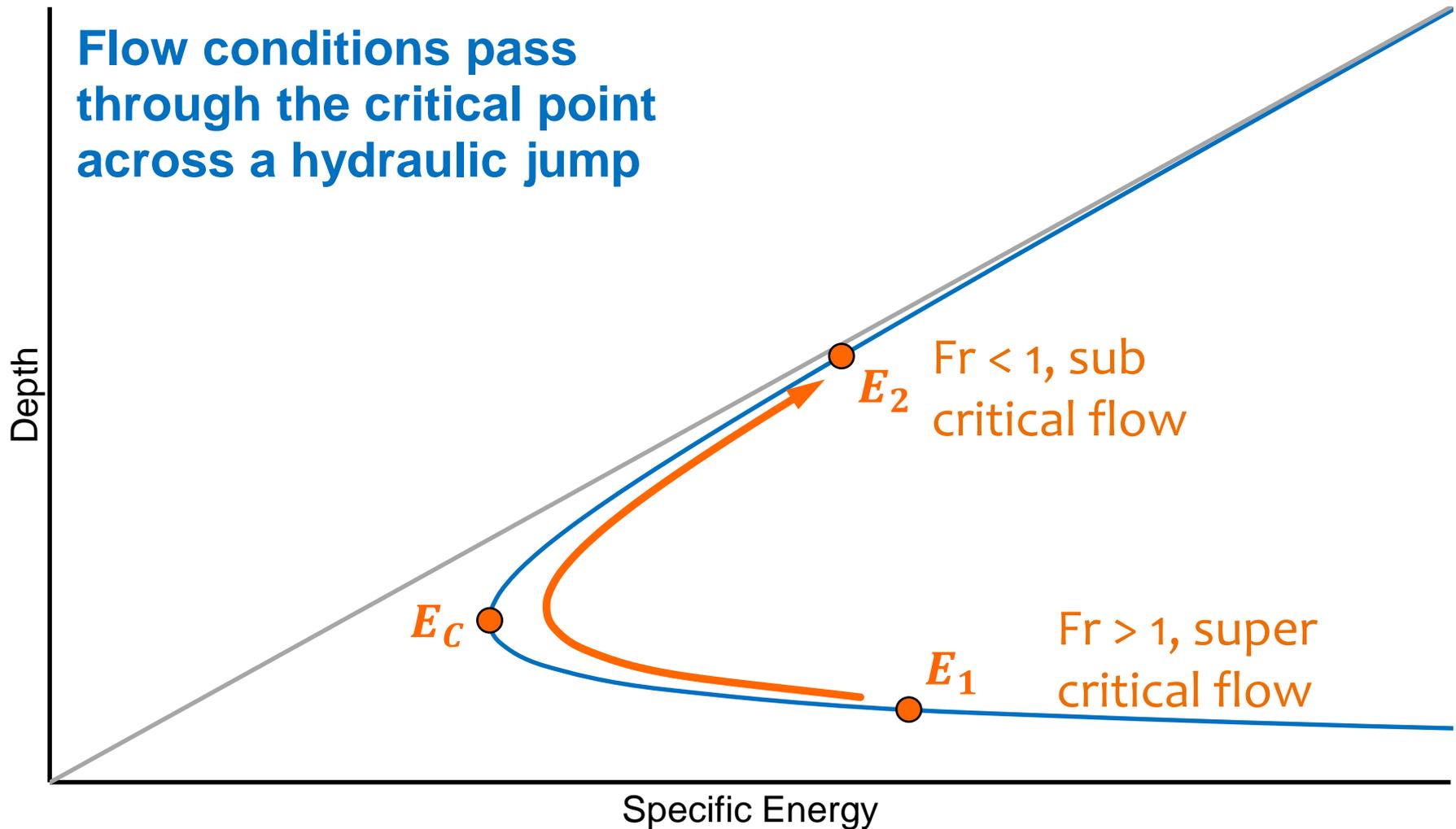

Fr < 1, normal depth established

Fr > 1

HIGH VELOCITY | HYDRAULIC JUMP TURBULENCE | SLOW VELOCITY

SUPERCRITICAL DEPTH | CRITICAL DEPTH | SUBCRITICAL DEPTH

Dept. Civil Eng., California State U. at Sacramento

Why is critical flow important?

Dept. of Civil Eng., University of Texas at Austin

Hydraulic Jumps

Flow conditions pass through the critical point across a hydraulic jump

E_2 Fr < 1, sub critical flow

E_C

E_1 Fr > 1, super critical flow

Depth

Specific Energy

Water Surface Profiles

- Under **gradually varied flow** conditions, flow depth is uneven along a channel

- Classify flow according to $y, y_n,$ and y_c:

| Name	ID	Channel Condition	Flow Condition
Mild	M	$y_n > y_c$	M1: $y > y_n$ M2: $y_n < y < y_c$ M3: $y < y_c$
Steep	S	$y_n < y_c$	S1: $y > y_c$ S2: $y_n < y < y_c$ S3: $y < y_n$
Critical	C	$y_n = y_c$	C1: $y > y_c$ C3: $y < y_c$
Horizontal	H	$y_n = \infty$	H2: $y > y_c$ H3: $y < y_c$
Adverse	A	$S_0 < 0$	A2: $y > y_c$ A3: $y < y_c$

Water Surface Profiles
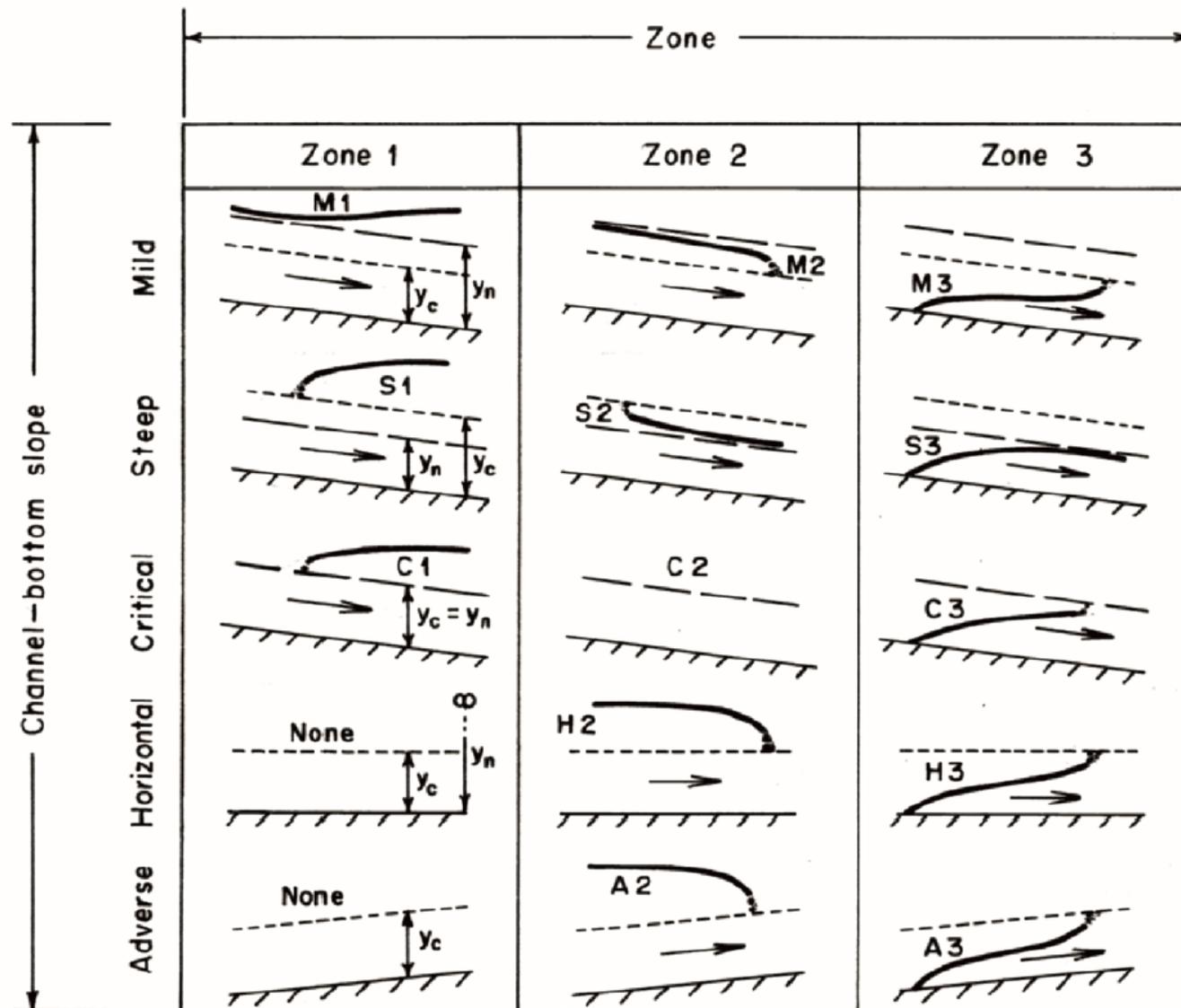
Example 2 – Water Surface Profiles

Problem: Classify and sketch the water surface profiles for the channel section shown below.
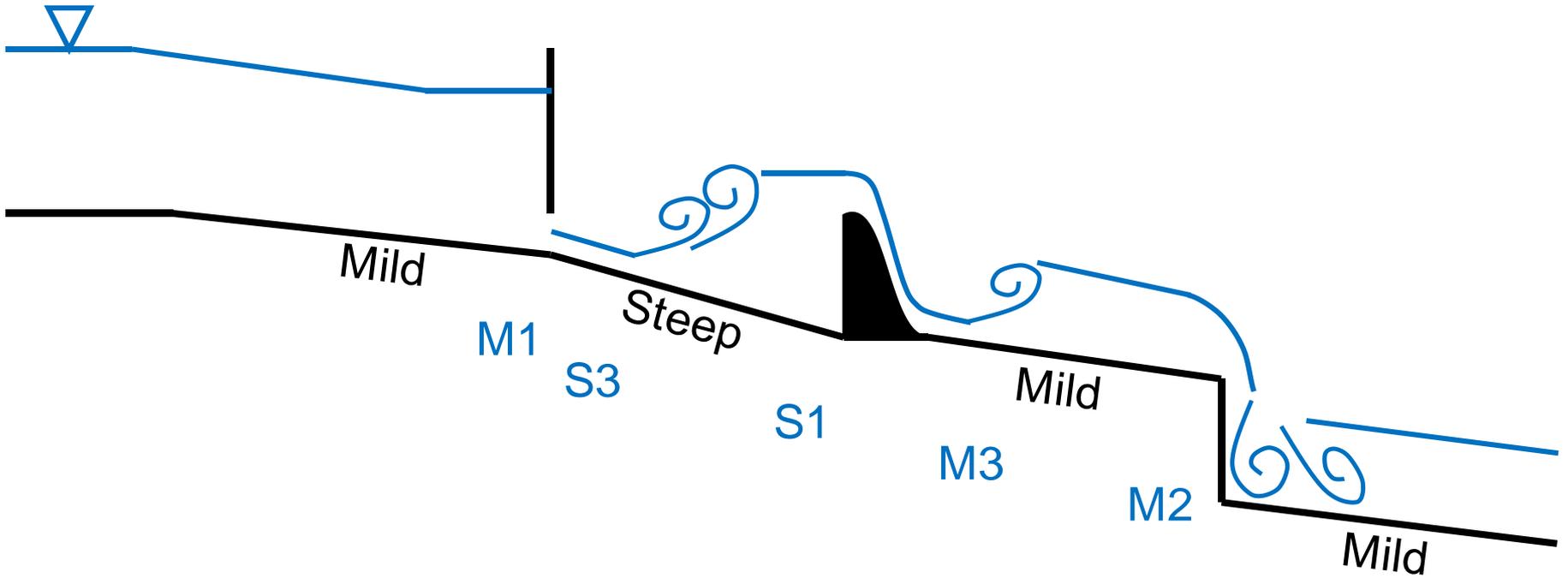
Example 2 – Water Surface Profiles

Problem: Classify and sketch the water surface profiles for the channel section shown below.

$y = y_c$

$y = y_c$

Mild
$y_n > y_c$

Steep
$y_n < y_c$

Mild
$y_n > y_c$

Mild
$y_n > y_c$
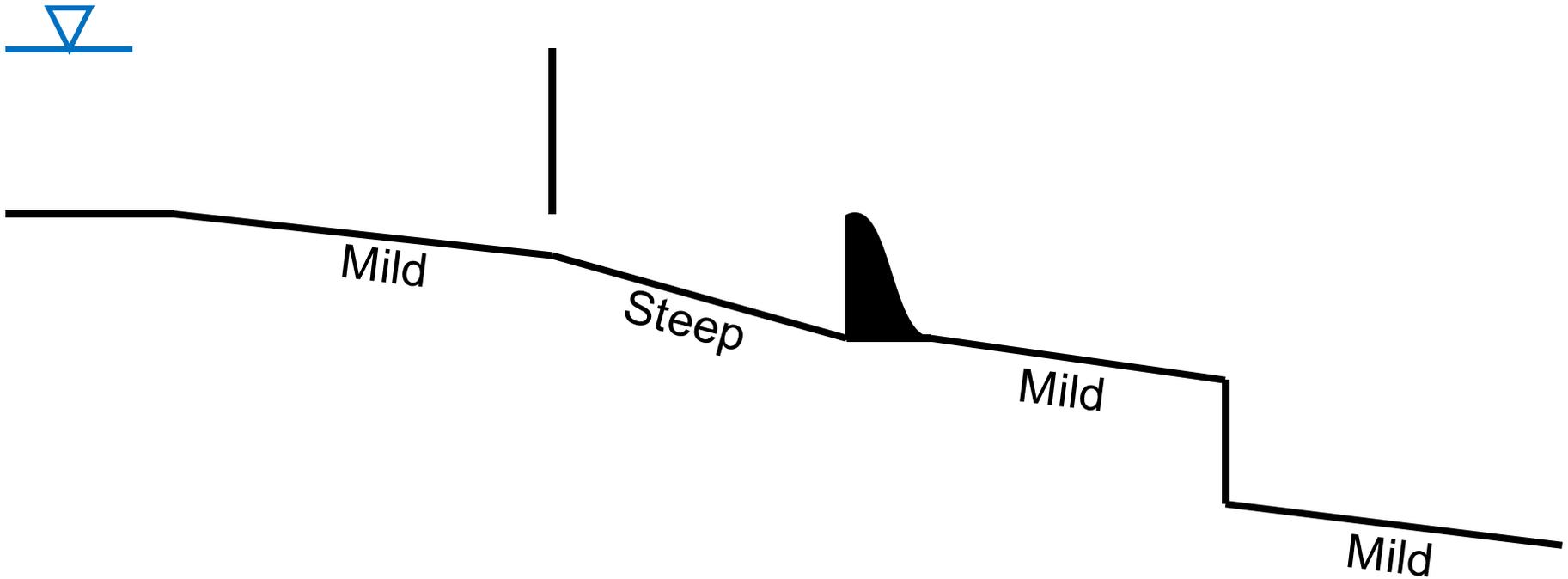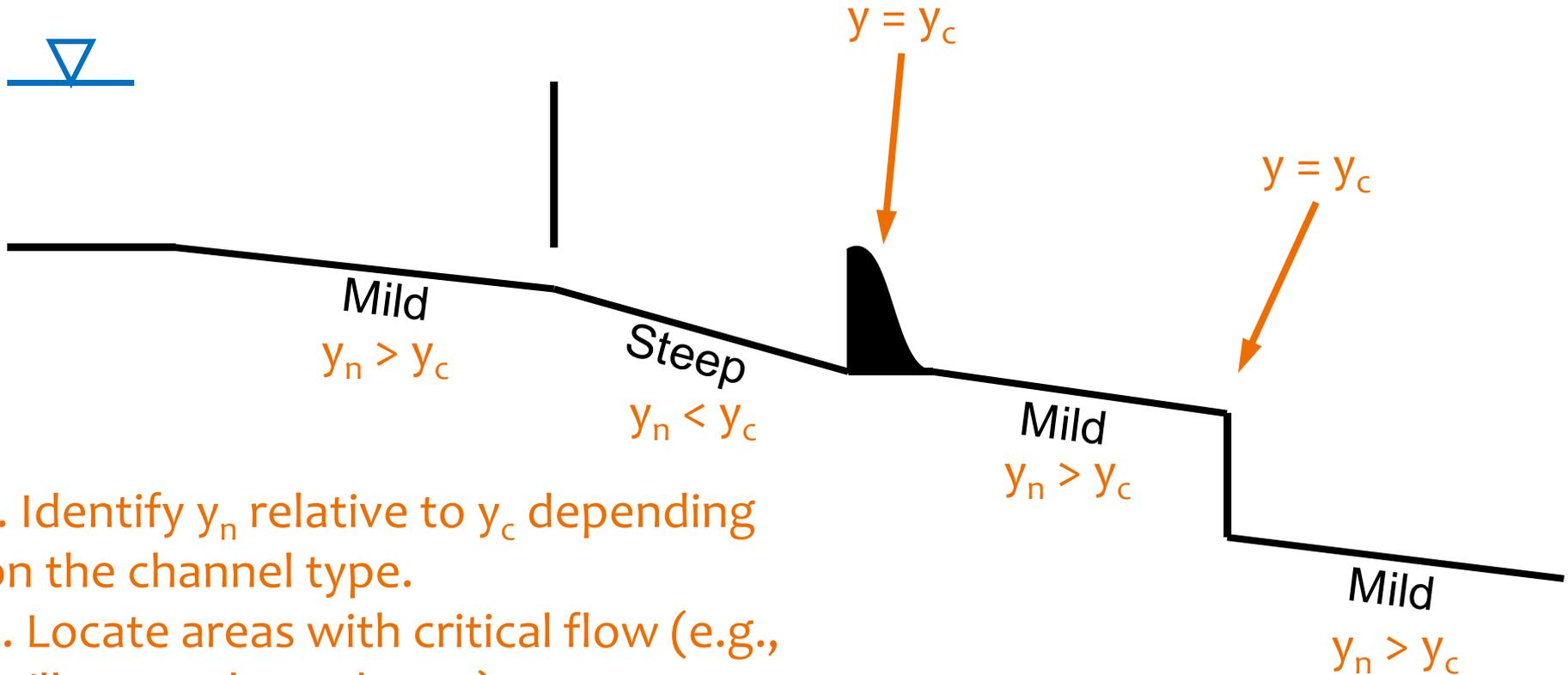
1. Identify y_n relative to y_c depending on the channel type.
2. Locate areas with critical flow (e.g., spillovers, along chutes)
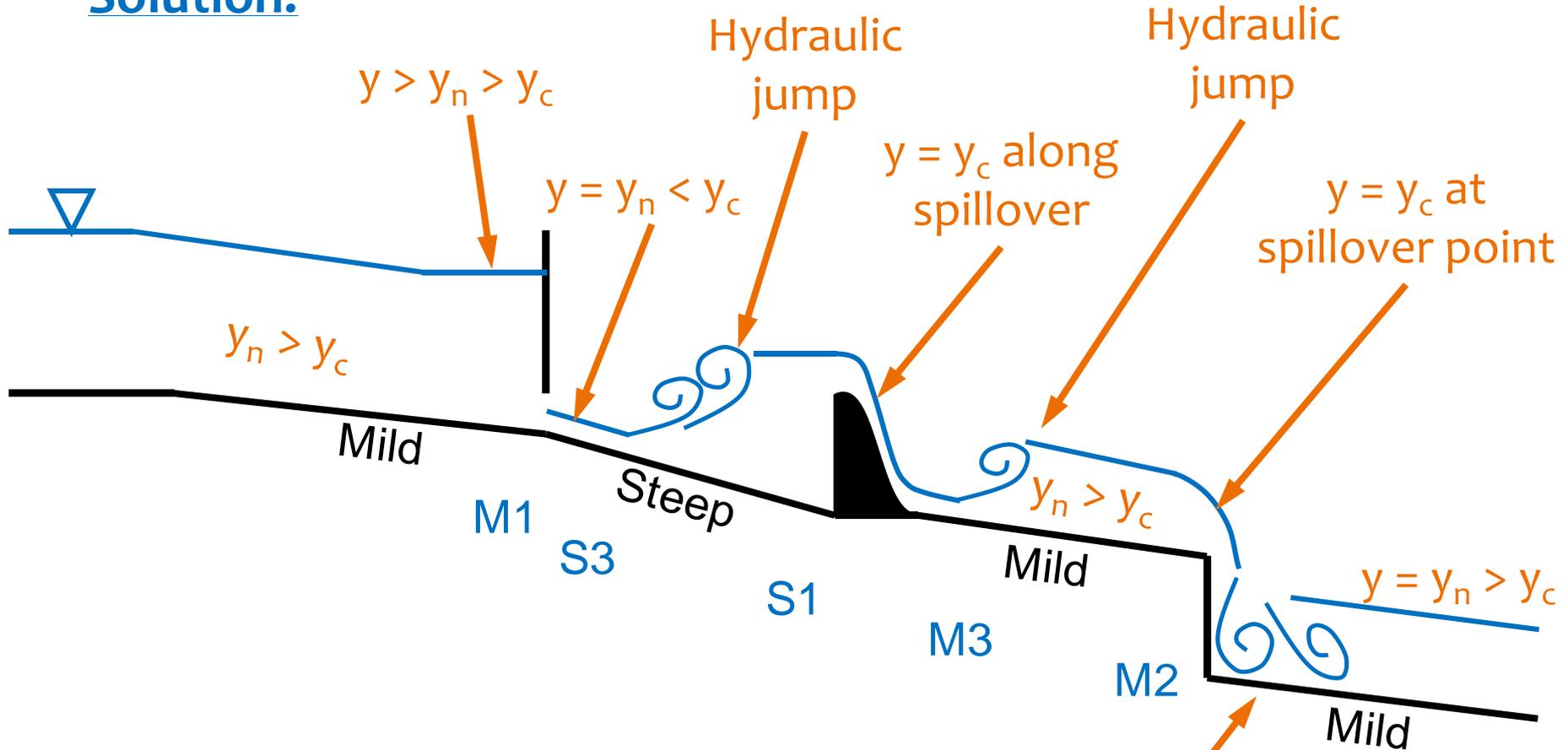3. Locate areas with hydraulic jumps.

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Example 2 – Water Surface Profiles

Solution:

Example 2 – Water Surface Profiles

Solution:

$y > y_n > y_c$

Hydraulic jump

Hydraulic jump

$y = y_n < y_c$

$y = y_c$ along spillover

$y = y_c$ at spillover point

$y_n > y_c$

$y_n > y_c$

Mild

M1

S3

Steep

S1

Mild

M3

$y = y_n > y_c$

M2

Mild

Depending on the downstream conditions, a hydraulic jump may or may not form

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